1. 希腊字母

Teorema 1
计算欧式看涨期权不支付红利下的敏感性度量（希腊字母）

proof. 由欧式看涨期权的价格公式：

$$C = SN(d_1) - Ke^{-r\tau}N(d_2)$$

其中：

$$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

1. Delta 值

   经济意义：期权价格对股价的敏感度，期权对冲时的股票数量

   数学意义：期权价格对股价的一阶导数

$$\begin{aligned} Delta = \frac{\partial C}{\partial S} &= N(d_1) + S\frac{\partial N(d_1)}{\partial S} - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial S} \\ &= N(d_1) + S\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}\frac{\partial d_1}{\partial S} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}}\frac{\partial d_2}{\partial S} \end{aligned}$$

   由 d_2 与 d_1 关系两边同时对 S 求偏导数可知：

$$Delta = N(d_1) + \frac{1}{\sqrt{2\pi}}\frac{\partial d_1}{\partial S}(Se^{-\frac{d_1^2}{2}} - Ke^{-r\tau}Se^{-\frac{d_2^2}{2}})$$

   由 d_1 的计算公式可知：

$$S = Ke^{d_1\sigma\sqrt{\tau} - (r + \frac{1}{2}\sigma^2)\tau} = Ke^{-r\tau}e^{d_1\sigma\sqrt{\tau} - \frac{1}{2}\sigma^2\tau} \qquad (*)$$

   即：

$$Se^{-\frac{d_1^2}{2}} = Ke^{-r\tau}e^{d_1\sigma\sqrt{\tau} - \frac{1}{2}\sigma^2\tau - \frac{d_1^2}{2}} = Ke^{-r\tau}e^{-\frac{(d_1 - \sigma\sqrt{\tau})^2}{2}}$$

   由 $d_2 = d_1 - \sigma\sqrt{\tau}$ 可知：

$$Se^{-\frac{d_1^2}{2}} = Ke^{-r\tau}e^{-\frac{d_2^2}{2}}$$

   将上式代入 $(*)$ 中可得：

$$Delta = N(d_1)$$

2. Gamma 值

   经济意义：Delta 对股价的敏感度，期权对冲时的股票数量变动的度量

数学意义：期权价格对股价的二阶导数

$$Gamma = \frac{\partial N(d_1)}{\partial S} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \frac{\partial d_1}{\partial S} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \frac{1}{S\sigma\sqrt{\tau}} > 0$$

则，欧式看涨期权的 $Gamma$ 恒为正数，这意味着为了平衡敞口，股价上涨时卖出更多；股价下跌时买入更多”

也就是，$Gamma > 0$ 时对应的平敞口操作是“高抛低吸”，而 $Gamma < 0$ 时对于的平敞口操作是“高买低卖”

3. Vega 值

   经济意义：期权对波动率的敏感度

   数学意义：期权价格对波动率的导数

$$\begin{aligned}
Vega = \frac{\partial C}{\partial \sigma} &= S\frac{\partial N(d_1)}{\partial \sigma} - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial \sigma} \\
&= S\frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \frac{\partial d_1}{\partial \sigma} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \frac{\partial d_2}{\partial \sigma} \\
&= S\frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} (\frac{\partial d_2}{\partial \sigma} + \sqrt{\tau}) - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \frac{\partial d_2}{\partial \sigma} \\
&= S\frac{\sqrt{\tau}}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}
\end{aligned}$$

4. Theta 值

   经济意义：期权对存续时间的敏感度

   数学意义：期权价格对存续时间的导数

$$\begin{aligned}
Theta = \frac{\partial C}{\partial \tau} &= S\frac{\partial N(d_1)}{\partial \tau} + rKe^{-r\tau}N(d_2) - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial \tau} \\
&= rKe^{-r\tau}N(d_2) + S\frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \frac{\partial d_1}{\partial \tau} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \frac{\partial d_2}{\partial \tau} \\
&= rKe^{-r\tau}N(d_2) + S\frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} (\frac{\partial d_2}{\partial \tau} + \frac{\sigma}{2\sqrt{\tau}}) - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \frac{\partial d_2}{\partial \tau} \\
&= rKe^{-r\tau}N(d_2) + S\frac{\sigma}{2\sqrt{2\pi\tau}} e^{-\frac{d_1^2}{2}}
\end{aligned}$$

5. Rho 值

   经济意义：期权对无风险收益率的敏感度

   数学意义：期权价格对无风险收益率的导数

$$\begin{aligned}
Rho = \frac{\partial C}{\partial r} &= S\frac{\partial N(d_1)}{\partial r} + \tau Ke^{-r\tau}N(d_2) - Ke^{-r\tau}\frac{\partial N(d_2)}{\partial r} \\
&= \tau Ke^{-r\tau}N(d_2) + S\frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \frac{\partial d_1}{\partial r} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \frac{\partial d_2}{\partial r} \\
&= \tau Ke^{-r\tau}N(d_2) + S\frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \frac{\partial d_2}{\partial r} - Ke^{-r\tau}\frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \frac{\partial d_2}{\partial r} \\
&= \tau Ke^{-r\tau}N(d_2)
\end{aligned}$$

对于欧式看跌期权的希腊字母，可以由看涨看跌平价关系推出：

$$P + S = C + Ke^{-r\tau}$$

即

$$P = C + Ke^{-r\tau} - S$$

□