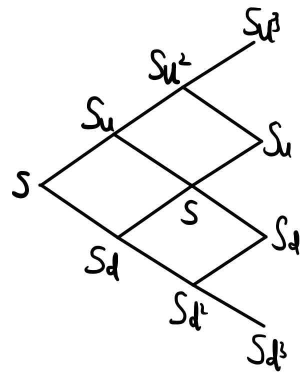
某股票价格为 90 美元， 利用三步二叉树计算期权价格： (a) 9 个月期限、 执行价格为 93 美元的 美式看涨期权； (b) 9 个月期限、执行价格为 93 美元的美式看跌期权。波动率为 28%, 无风险利率（所有期限） 为 3% (连续复利）。

即Δt=三个月=0.25年



解：由题 $\Delta t = \frac{1}{4}$，$r = 3\%$，$\sigma = 28\%$

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.28\times\sqrt{\frac{1}{4}}} \approx 1.150 \quad , \quad d = \frac{1}{u} \approx 0.869$$

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.03\times\frac{1}{4}} - d}{u - d} \approx 0.493$$

则有：$Su^3 = u^3 \cdot S_0 = 1.15^3 \times 90 = 136.879$ $\qquad Sd^3 = d^3 \cdot S_0 = 0.869^3 \times 90 = 59.061$

$Su^2 = u^2 \cdot S_0 = 1.15^2 \times 90 = 119.025$ $\qquad Sd^2 = d^2 \cdot S_0 = 0.869^2 \times 90 = 67.964$

$Su = u \cdot S_0 = 1.15 \times 90 = 103.5$ $\qquad Sd = d \cdot S_0 = 0.869 \times 90 = 78.21$

又由上涨与下跌幅度不变，故有 $Sud = Sdu = S = 90$

$Su^2d = Su = 103.5 \qquad Sud^2 = Sd = 78.21$

则可利用倒退定价法推算期权价值：

(a) 对于 $K = 93$ 的 9个月美式看涨期权有：

$$f_{u^3} = \max\{0, Su^3 - K\} = \max\{0, 136.879 - 93\} = 43.879$$

$$f_{u^2d} = \max\{0, Su^2d - K\} = \max\{0, 103.5 - 93\} = 10.5$$

$$f_{ud^2} = \max\{0, Sud^2 - K\} = \max\{0, 78.21 - 93\} = 0$$

$$f_{d^3} = \max\{0, Sd^3 - K\} = \max\{0, 59.061 - 93\} = 0$$

↓

$$f_{u^2} = \max\{Su^2 - K, e^{-0.03\times\frac{1}{4}}(0.493\times 43.879 + 0.507\times 10.5)\} = 26.754$$

$$f_{ud} = \max\{Sud - K, e^{-0.03\times\frac{1}{4}}(0.493\times 10.5 + 0.507\times 0)\} = 5.138$$

$$f_{d^2} = \max\{Sd^2 - K, 0\} = 0$$

↓

$f_u = \max\{S_u - K, e^{-0.03\times\frac{1}{4}}(0.493\times 26.754 + 0.507\times 5.138)\} = 15.677$

$f_d = \max\{S_d - K, e^{-0.03\times\frac{1}{4}}(0.493\times 5.138 + 0.507\times 0)\} = 2.514$

$\downarrow$

$f = e^{-0.03\times\frac{1}{4}}(0.493\times 15.677 + 0.507\times 2.514) = 8.936.$

(b) 对于 K=93 的 9 个月美式看跌期权:

$f_{u^3} = \max\{0, K - S_{u^3}\} = \max\{0, -43.879\} = 0$

$f_{u^2d} = \max\{0, K - S_{u^2d}\} = \max\{0, -10.5\} = 0$

$f_{ud^2} = \max\{0, K - S_{ud^2}\} = \max\{0, 14.79\} = 14.79$

$f_{d^3} = \max\{0, K - S_{d^3}\} = \max\{0, 33.939\} = 33.939$

$\downarrow$

$f_{u^2} = \max\{K - S_{u^2}, e^{-0.03\times\frac{1}{4}}(0.493\times 0 + 0.507\times 0)\} = 0$

$f_{ud} = \max\{K - S_{ud}, e^{-0.03\times\frac{1}{4}}(0.493\times 0 + 0.507\times 14.79)\} = 7.443$

$f_{d^2} = \max\{K - S_{d^2}, e^{-0.03\times\frac{1}{4}}(0.493\times 14.79 + 0.507\times 33.939)\} = 25.036$

$\downarrow$

$f_u = \max\{K - S_u, e^{-0.03\times\frac{1}{4}}(0.493\times 0 + 0.507\times 7.443)\} = 3.745$

$f_d = \max\{K - S_d, e^{-0.03\times\frac{1}{4}}(0.493\times 7.443 + 0.507\times 25.036)\} = 16.24$

$\downarrow$

$f = e^{-0.03\times\frac{1}{4}}(0.493\times 3.745 + 0.507\times 16.24) = 10.005$